Since AP Calculus BC has two pathways into the course (Advanced Precalculus/Calculus or AP Calculus AB), the requirements for this packet are different depending on which course you have just completed. This packet is designed to provide students coming from the Advanced Precalculus/Calculus course with exposure to a few topics that students from AP Calculus AB have already studied.

If you are coming from the Advanced Precalculus/Calculus course:

It is STRONGLY recommended that you complete this packet over the summer to gain exposure to topics that will be needed for next year. That way you will have fewer topics that you have never seen compared to those coming from AP Calculus AB. The key word is exposure. It is NOT expected that you will have complete mastery of these topics simply from completing this packet. Each of these topics will be covered within the course next year. The next page will provide more details.

If you are coming from AP Calculus AB:

This packet is optional. If you look at the topics covered (Basic Integration rules, Riemann Sums, and Slope Fields) and you are confident that you have mastered those topics, then you do not need to complete the packet. If there are one or more topics in this packet that you struggled with in AP Calculus AB, then I would strongly recommend completing just that portion of this packet. Either way I do suggest that over the summer, especially in August, you look over your notes and work from AP Calculus AB to prepare for AP Calculus BC.

Congratulations! You've made it through the first year! Since you will now have to take an AP exam by the end of this next year, it is VITAL that you let me know when you need help **as soon as** you need it. Especially as we get towards the middle of the year, there will be topics that may be more of a challenge for you than students who went through the whole year of AB calculus. I cannot provide you the extra support you may need UNLESS you let me know that you are struggling with the topic. So please **do not be afraid to ask**. I want **everyone** to be successful in the BC course, regardless of the path taken to get there. **I cannot help you get past a struggle if I do not know you are struggling.**

Within this packet I have tried to focus in on a few key areas, as suggested by the first few years of students to go through the program. These topics are not very difficult once you get the hang of them, and as such we go through them relatively quickly within the BC course. So hopefully with this added practice you will feel more comfortable with the topics when you see them again in the BC course. For each topic, you can read through the examples provided to see how the topic works, and then try the practice problems. A few of the practice problems will have answers provided so that you can self-check your steps before you complete the rest of the problems for that topic.

If you have any questions at all while you are working through the packet, feel free to email me at aferreira@lowell.k12.ma.us . Towards the end of June, I will set up a Google classroom with a slideshow that walks you through the examples step by step with added visuals, so if you do not see that from me by July 1st, please email me to let me know.

I hope you have a wonderful summer! To make your chances for success as high as possible, in addition to completing this packet, in August you should also go over all of your Calculus notes from this past semester and review your trigonometry and key algebra skills (factoring, solving any type of equation, log rules, domain restrictions). The predominant struggles in BC stem from not knowing trigonometry as well as one should and algebra mistakes more than the actual calculus itself.

This past semester we spent a lot of time looking at the topic of differentiation. One of the first main new topics that you will see in the BC curriculum is antidifferentiation.

Antidifferentiation is the inverse process of differentiation. For instance, we know that the derivative of $y = x^3$ is $3x^2$. So it should make sense to say that an antiderivative of $3x^2$ is x^3. However, it is important to say *an* antiderivative of $3x^2$ rather than *the* antiderivative of $3x^2$ for the simple reason that the derivative of $y = x^3$ is $3x^2$, but so is the derivative of $y = x^3 + 2$, $y = x^3 - 5$, and $y = x^3 + 6\pi$ (and so on for any $y = x^3 \pm$ some constant). So when we go backwards to the antiderivative it is impossible to determine which exact function it came from without more information about the original function. So to cover ourselves for whatever that constant may be, we say that the antiderivative of $3x^2$ is $y = x^3 + C$, where *C* represents a constant. We call *C* the **constant of integration**. [AP Exam note: one point in the free response questions involving antiderivatives is given simply for writing the "+*C*", and no other points are given after that if it is forgotten, so it is VITAL to make sure you don't forget it!]

The process of taking antiderivatives is called **integration**, specifically **indefinite integration** because of the constant of integration *C*. For the differentiation process we have the notation $\frac{d}{dx}(f(x))$ to derive some function $f(x)$. For the antidifferentiation process we use the $\int$ symbol to integrate. The antidifferentiation process has the notation $\int f(x)dx$ to integrate some function $f(x)$. The *dx* tells you what the important variable is when you are integrating – what you are integrating in terms of.

Just as we had differentiation rules to memorize, we have corresponding integration rules that **MUST** be **MEMORIZED**.

Differentiation formula	*Integration formula*
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$ a constant can be "factored out"	$\int kf(x)\,dx = k\int f(x)\,dx + C$ - factor out constant
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$ the derivative of a sum is the sum of derivatives	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx + C$ integral of a sum is the sum of the integrals
$\frac{d}{dx}[x^n] = nx^{n-1}$ - the power rule	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ - the power rule reversed

Differentiation formula	Integration formula
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2\ dx = -\cot x + C$

Within the BC course we'll take a look together at compensating for Chain Rule with integrals, and learn the other methods of integration for when it does not match one of these basic rules from the charts. But all of that work will seem easier if you can get extra practice with the basic rules to start.

<u>**Example #1 – Indefinite integration**</u>

(a) $\int 7dx$

$= 7x + C$

(b) $\int x^5 dx$

$= \frac{x^6}{6} + C$

(c) $\int (3x^4 - 3x^2)dx$

$= \frac{3x^5}{5} - x^3 + C$

(d) $\int \frac{1}{x^3} dx$

$= \int x^{-3} dx$

$= \frac{x^{-2}}{-2} + C = -\frac{1}{x^2} + C$

(e) $\int \sqrt[3]{x^2} dx$

$= \int x^{2/3} dx$

$= \frac{3}{5} x^{5/3} + C$

$= \frac{3}{5} \sqrt[3]{x^5} + C$

(f) $\int \left(\frac{1}{\sqrt{x}} - x^{3/4}\right) dx$

$= \int (x^{-1/2} - x^{3/4}) dx$

$= \frac{1}{2} x^{1/2} - \frac{4}{7} x^{7/4} + C$

$= \frac{1}{2} \sqrt{x} - \frac{4}{7} x^{7/4} + C$

NOTE: You should ALWAYS be able to check your integration answers by taking the derivative of your answer and making sure you get what's in the integral you started with.

Until we learn some of the other methods and how to compensate for Chain Rule, here are a few tricks to get back to basic rules:

Example #2 – Indefinite integration tricks

(a) $\int (2x-3)^2 dx$

Multiply it out.

$= \int (4x^2 - 12x + 9) dx$

$= \frac{4}{3}x^3 - 6x^2 + 9x + C$

(b) $\int \frac{x^2+3x+1}{x^4} dx$

Split into individual fractions and integrate.

$= \int \left(\frac{1}{x^2} + \frac{3}{x^3} + \frac{1}{x^4}\right) dx$

$= \frac{-1}{x} - \frac{3}{2x^2} - \frac{1}{5x^5} + C$

(c) $\int \frac{(2x-5)(3x+2)}{\sqrt{x}} dx$

$= \int \frac{6x^2-11x-10}{x^{1/2}} dx$

$= \int \left(6x^{\frac{3}{2}} - 11x^{\frac{1}{2}} - 10x^{-\frac{1}{2}}\right) dx$

$= \frac{12}{5}x^{\frac{5}{2}} - \frac{22}{3}x^{\frac{3}{2}} - 20x^{\frac{1}{2}} + C$

Example #3 – Indefinite integration with trigonometric functions

(a) $\int 4 \sin x \, dx$

$= -4 \cos x + C$

(b) $\int \frac{-2 \cos x}{3} dx$

$= \int -\frac{2}{3} \cos x \, dx$

$= -\frac{2}{3} \sin x + C$

(c) $\int \frac{5}{\cos^2 x} dx$

$= \int 5 \sec^2 x \, dx$

$= 5 \tan x + C$

(d) $\int (4 \cos x - 9 \sin x) dx$

$= 4 \sin x + 9 \cos x + C$

(e) $\int \frac{-\sin x}{\cos^2 x} dx$

$= \int -\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$

$= \int -\tan x \sec x \, dx$

$= -\sec x + C$

(f) $\int (x^2 - 2 \csc^2 x) dx$

$= \frac{x^3}{3} + 2 \cot x + C$

Indefinite Integration PRACTICE

1. $\int -9\,dx$

2. $\int -5x\,dx$

3. $\int (6+2x)\,dx$

4. $\int x^7\,dx$

5. $\int (x^4+x^3-x^2)\,dx$

6. $\int (3x^3-4x^2)\,dx$

7. $\int \left(\frac{2}{3}x^5-\frac{5}{2}x+\frac{1}{2}\right)dx$

8. $\int \left(\frac{3}{x^4}\right)dx$

9. $\int \left(2-\frac{1}{x^5}+\frac{7}{x^3}\right)dx$

10. $\int 5\sqrt{x}\,dx$

11. $\int 5\left(\sqrt[5]{x}\right)dx$

12. $\int \left(x^{3/4}-\frac{1}{x^{3/4}}\right)dx$

13. $\int 3\sqrt[3]{x^2}\,dx$

14. $\int (x-5)^2\,dx$

15. $\int 4(3x-2)^3\,dx$

16. $\int \frac{x^3-4x-1}{2x^3}\,dx$

17. $\int t^2(3+t)^2\,dt$

18. $\int \frac{(3x-2)^2}{\sqrt{x}}\,dx$

19. $\int \frac{3\cos x}{5}\, dx$

20. $\int (1 - 6\cos x)\, dx$

21. $\int \left(\frac{1}{x^2} - \sin x\right) dx$

22. $\int \left(\sec^2 t + \cos t + 1\right) dt$

23. $\int \left(\sin^2 x + \cos^2 x\right) dx$

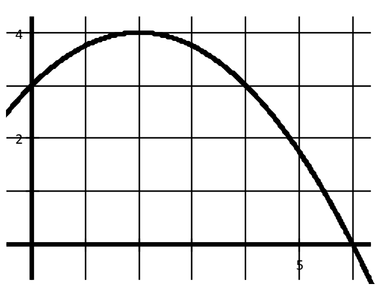
Hint: What trig identity can make this MUCH easier?

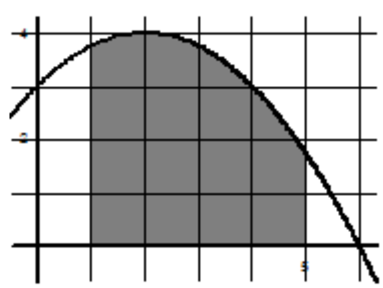
24. $\int \frac{\sin x}{1 - \sin^2 x}\, dx$

Hint: Use a trig identity to eliminate subtraction in denom.

The integral of a function represents the area between the curve and the *x*-axis, and finding this area is another common problem in calculus. A way to estimate the area under the curve is by **Riemann sums** and **Trapezoidal sums**. This is especially helpful when you cannot find the expression for the integral and/or you only have the graph of the function or a table of a few values.

As a common example, this next topic will use this problem: find the area under the function $f(x)$, given in the picture below to the left, from $x = 1$ to $x = 5$. What we are looking for is the picture below on the right.

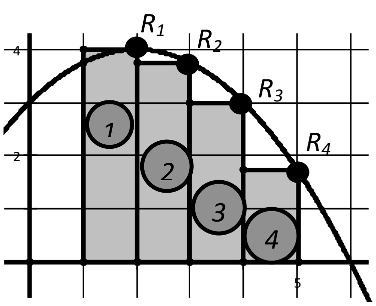




We will look at four techniques to estimate this area: right rectangles, left rectangles, midpoint rectangles, and trapezoids. You can use any number of rectangles or trapezoids to estimate the area, and the more of them you use the more accurate your approximation will be. [AP Exam Note: Any Riemann sum calculated is an <u>**approximation**</u> and, like the derivative approximations, will require the ≈ symbol to earn your answer point on the AP exam.] The **first step** in any Riemann sum problem is to figure out **how many rectangles or trapezoids** you will be using to break down and approximate the area. The **next step** will be to figure out the **width of each of the rectangles** or the **height of each trapezoid**. Then depending on which type of sum you are setting up, you can find the lengths for each rectangle and the bases for each trapezoid.

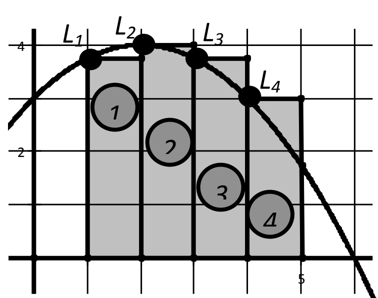
For the problem given to start, we will first approximate the area by using four rectangles or four trapezoids. This makes our work easier, because if we break down from $x = 1$ to $x = 5$ into 4 pieces ($n = 4$), each piece will be one unit wide: $\left(\frac{x_2 - x_1}{n}\right) = \left(\frac{5-1}{4}\right) = 1$.

RIGHT RECTANGLES



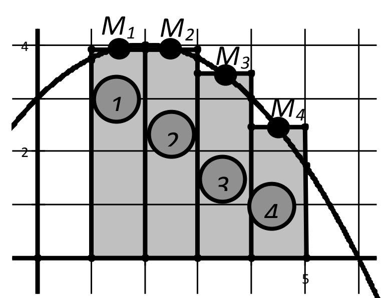
Each rectangle has height that corresponds to the **right** endpoint of the interval. In this instance, you can see that more of the actual area is missing than what extends over the curve – so this would be an **underestimate**. NOTE: <u>NOT</u> all right sums are underestimates.

LEFT RECTANGLES



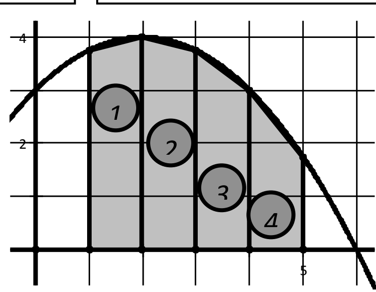
Each rectangle has height that corresponds to the **left** endpoint of the interval. In this instance, you can see that more of the estimate area extends over the curve than actual area missing – so this would be an **overestimate**. NOTE: <u>NOT</u> all left sums are overestimates.

MIDPOINT RECTANGLES



Each rectangle has height that corresponds to the **midpoint** of the interval. This is the closest rectangle approximation to the actual area.

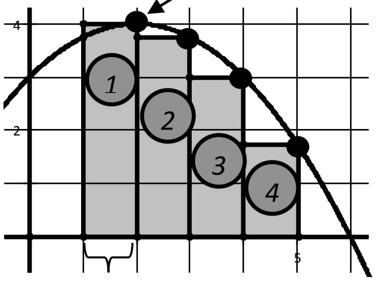
TRAPEZOIDAL SUMS



Each trapezoid has height along the *x*-axis and one base is the length up to the left endpoint and one base the length up to the right endpoint of each interval. You can see that clearly this is the closest (best) approximation.

RIGHT RECTANGLES

Each height is the function value, *f*(*x*), of each right endpoint.



Each width = 1

$A \approx R_4 =$ Area of 1 + Area of 2 + Area of 3 + Area of 4

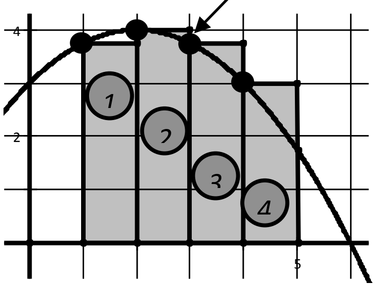
$A \approx R_4 = w_1 \cdot h_1 + w_2 \cdot h_2 + w_3 \cdot h_3 + w_4 \cdot h_4$

$A \approx R_4 = (1)\big(f(2)\big) + (1)\big(f(3)\big) + (1)\big(f(4)\big) + (1)\big(f(5)\big)$

$A \approx R_4 = f(2) + f(3) + f(4) + f(5)$

LEFT RECTANGLES

Each height is the function value, *f*(*x*), of each left endpoint.



$A \approx L_4 =$ Area of 1 + Area of 2 + Area of 3 + Area of 4

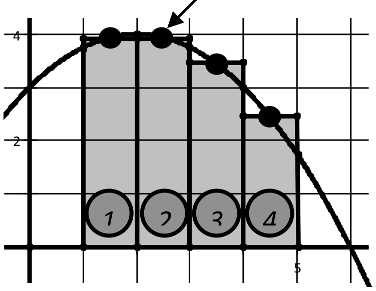
$A \approx L_4 = w_1 \cdot h_1 + w_2 \cdot h_2 + w_3 \cdot h_3 + w_4 \cdot h_4$

$A \approx L_4 = (1)\big(f(1)\big) + (1)\big(f(2)\big) + (1)\big(f(3)\big) + (1)\big(f(4)\big)$

$A \approx L_4 = f(1) + f(2) + f(3) + f(4)$

MIDPOINT RECTANGLES

Each height is the function value, *f*(*x*), of the midpoint of each interval.



$A \approx M_4 =$ Area of 1 + Area of 2 + Area of 3 + Area of 4

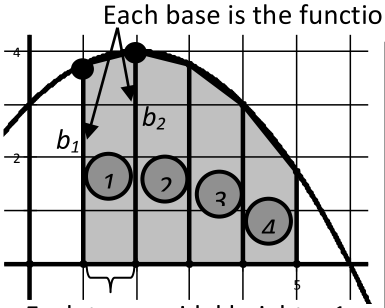
$A \approx M_4 = w_1 \cdot h_1 + w_2 \cdot h_2 + w_3 \cdot h_3 + w_4 \cdot h_4$

$A \approx M_4 = (1)\big(f(1.5)\big) + (1)\big(f(2.5)\big) + (1)\big(f(3.5)\big) + (1)\big(f(4.5)\big)$

$A \approx M_4 = f(1.5) + f(2.5) + f(3.5) + f(4.5)$

TRAPEZOIDAL SUMS

Each base is the function value, *f*(*x*), on either side of each interval.



Each trapezoidal height = 1

$A \approx T_4 =$ Area of 1 + Area of 2 + Area of 3 + Area of 4

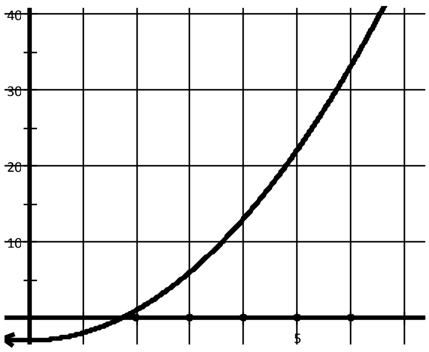
$$A \approx T_4 = \frac{h_1(b_1+b_2)}{2} + \frac{h_2(b_2+b_3)}{2} + \frac{h_3(b_3+b_4)}{2} + \frac{h_4(b_4+b_5)}{2}$$

$$A \approx T_4 = \frac{(1)\big(f(1)+f(2)\big)}{2} + \frac{(1)\big(f(2)+f(3)\big)}{2} + \frac{(1)\big(f(3)+f(4)\big)}{2} + \frac{(1)\big(f(4)+f(5)\big)}{2}$$

$$A \approx T_4 = \frac{1}{2}[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)]$$

Ex. 4 – Riemann sums using equation

Let $f(x) = x^2 - 3$. We want to estimate the area under the curve using 8 rectangles/ trapezoids from *x* = 2 to *x* = 6. Although it's not necessary, we can take a look at the graph.



So we know to start we want 8 rectangles or trapezoids. If they are to be evenly spaced, next we can figure out the width of each one: $\frac{x_2 - x_1}{n} = \frac{6-2}{8} = \frac{4}{8} = \frac{1}{2}$. So each rectangle will have width of $\frac{1}{2}$. Each trapezoid will have height of $\frac{1}{2}$.

From here it can be helpful to make a list or table of values so we know the heights for the rectangles and the bases for the trapezoids. For the *x*-values, we start at 2 and go by $\frac{1}{2}$ until we reach 6. Then fill in the corresponding *y*-values using the function. (You may use fractions or decimals, whichever seems easiest to you.) Note: If you use decimals, it must be AT LEAST 5 decimal places used so that FINAL answer is accurate to three decimal places.

x	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	1	3.25	6	9.25	13	17.25	22	27.25	33

RIGHT RECTANGLES

		h of R_1	h of R_2	h of R_3	h of R_4	h of R_5	h of R_6	h of R_7	h of R_8
x	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	1	3.25	6	9.25	13	17.25	22	27.25	33

$$A \approx R_8 = \left(\tfrac{1}{2}\right) f(2.5) + \left(\tfrac{1}{2}\right) f(3) + \left(\tfrac{1}{2}\right) f(3.5) + \left(\tfrac{1}{2}\right) f(4) + \left(\tfrac{1}{2}\right) f(4.5) + \left(\tfrac{1}{2}\right) f(5) + \left(\tfrac{1}{2}\right) f(5.5) + \left(\tfrac{1}{2}\right) f(6)$$

$$A \approx R_8 = \left(\tfrac{1}{2}\right)(3.25) + \left(\tfrac{1}{2}\right)(6) + \left(\tfrac{1}{2}\right)(9.25) + \left(\tfrac{1}{2}\right)(13) + \left(\tfrac{1}{2}\right)(17.25) + \left(\tfrac{1}{2}\right)(22) + \left(\tfrac{1}{2}\right)(27.25) + \left(\tfrac{1}{2}\right)(33)$$

$$A \approx R_8 = 65.5$$

LEFT RECTANGLES

	h of L_1	h of L_2	h of L_3	h of L_4	h of L_5	h of L_6	h of L_7	h of L_8	
x	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	1	3.25	6	9.25	13	17.25	22	27.25	33

A ≈ L_8 $=\left(\frac{1}{2}\right)f(2)+\left(\frac{1}{2}\right)f(2.5)+\left(\frac{1}{2}\right)f(3)+\left(\frac{1}{2}\right)f(3.5)+\left(\frac{1}{2}\right)f(4)+\left(\frac{1}{2}\right)f(4.5)+\left(\frac{1}{2}\right)f(5)+\left(\frac{1}{2}\right)f(5.5)$

A ≈ L_8 $=\left(\frac{1}{2}\right)(1)+\left(\frac{1}{2}\right)(3.25)+\left(\frac{1}{2}\right)(6)+\left(\frac{1}{2}\right)(9.25)+\left(\frac{1}{2}\right)(13)+\left(\frac{1}{2}\right)(17.25)+\left(\frac{1}{2}\right)(22)+\left(\frac{1}{2}\right)(27.25)$

A ≈ L_8 = 49.5

MIDPOINT RECTANGLES

Midpoint rectangles are more complicated because we must make a new table with the midpoints of each interval from the original table. For instance, for the first midpoint it will be $\frac{2+2.5}{2}=2.25$, the next $\frac{2.5+3}{2}=2.75$, and so on.

	h of M_1	h of M_2	h of M_3	h of M_4	h of M_5	h of M_6	h of M_7	h of M_8
x	2.25	2.75	3.25	3.75	4.25	4.75	5.25	5.75
$f(x)$	$33/16$	$73/16$	$121/16$	$177/16$	$241/16$	$313/16$	$393/16$	$481/16$

A ≈ L_8

$=\left(\frac{1}{2}\right)f(2.25)+\left(\frac{1}{2}\right)f(2.75)+\left(\frac{1}{2}\right)f(3.25)+\left(\frac{1}{2}\right)f(3.75)+\left(\frac{1}{2}\right)f(4.25)+\left(\frac{1}{2}\right)f(4.75)+$
$\left(\frac{1}{2}\right)f(5.25)+\left(\frac{1}{2}\right)f(5.75)$

A ≈ L_8 $=\left(\frac{1}{2}\right)\left(\frac{33}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{73}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{121}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{177}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{241}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{313}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{393}{16}\right)+\left(\frac{1}{2}\right)\left(\frac{481}{16}\right)$

A ≈ L_8 = 57.25

TRAPEZOIDAL SUM

	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
x	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	1	3.25	6	9.25	13	17.25	22	27.25	33

$$A \approx T_8 = \frac{(h)(b_1+b_2)}{2} + \frac{(h)(b_2+b_3)}{2} + \frac{(h)(b_3+b_4)}{2} + \frac{(h)(b_4+b_5)}{2} + \frac{(h)(b_5+b_6)}{2} + \frac{(h)(b_6+b_7)}{2} + \frac{(h)(b_7+b_8)}{2} + \frac{(h)(b_8+b_9)}{2}$$

$$A \approx T_8 = \frac{\left(\frac{1}{2}\right)(f(2)+f(2.5))}{2} + \frac{\left(\frac{1}{2}\right)(f(2.5)+f(3))}{2} + \frac{\left(\frac{1}{2}\right)(f(3)+f(3.5))}{2} + \frac{\left(\frac{1}{2}\right)(f(3.5)+f(4))}{2} + \frac{\left(\frac{1}{2}\right)(f(4)+f(4.5))}{2} + \frac{\left(\frac{1}{2}\right)(f(4.5)+f(5))}{2} + \frac{\left(\frac{1}{2}\right)(f(5)+f(5.5))}{2} + \frac{\left(\frac{1}{2}\right)(f(5.5)+f(6))}{2}$$

$$A \approx T_8 = \frac{(1+3.25)}{4} + \frac{(3.25+6)}{4} + \frac{(6+9.25)}{4} + \frac{(9.25+13)}{4} + \frac{(13+17.25)}{4} + \frac{(17.25+22)}{4} + \frac{(22+27.25)}{4} + \frac{(27.25+33)}{4}$$

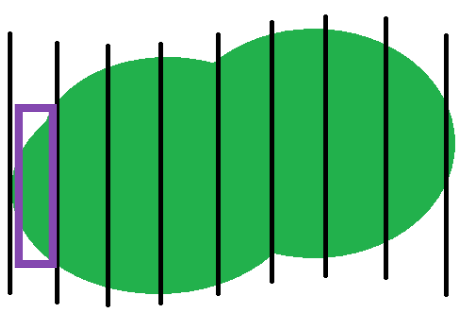
$$A \approx T_8 = 57.5$$

Ex. 5 – Riemann Sums using table of values

Cedarbrook golf course is constructing a new green. To estimate the area of the green, the caretaker draws parallel lines 10 feet apart and then measures the length of the green along that line. Determine how many square feet of grass sod must be purchased to cover the green if:

(a) The caretaker is lazy and uses 4 midpoint rectangles to calculate the area.
(b) The caretaker uses 8 left rectangles to calculate the area.
(c) The caretaker uses 8 right rectangles to calculate the area.
(d) The caretaker uses 8 trapezoids to calculate the area.

Length (ft)	0	28	50	62	60	55	51	30	3



Just so you get the basic idea, you can imagine the golf green sliced into strips 10 feet apart. The 10 feet apart will be one dimension for our estimate and we'll use the distance at each line measured as the other dimension for our estimates. The first right rectangle is drawn in the diagram (width 10, height 28).

(a) For the midpoint sum, since we are only using 4 rectangles, each width will be 20 ft.

		h of M_1		h of M_2		h of M_3		h of M_4	
Length (ft)	0	28	50	62	60	55	51	30	3

$$A \approx M_4 = (20)(28) + (20)(62) + (20)(55) + (20)(30) = 3500\ ft^2$$

$$A \approx 3500\ ft^2$$

(b) For the left sum, we are using 8 rectangles, each width will be 10 ft. (The 10 feet marker lines split the area into 8 rectangles.)

	h of L_1	h of L_2	h of L_3	h of L_4	h of L_5	h of L_6	h of L_7	h of L_8	
Length (ft)	0	28	50	62	60	55	51	30	3

$$A \approx L_8 = (10)(0) + (10)(28) + (10)(50) + (10)(62) + (10)(60) + (10)(55) + (10)(51) + (10)(30) = 3360\ ft^2$$

$$A \approx 3360\ ft^2$$

(c) For the right sum, we are using 8 rectangles, each width will be 10 ft.

		h of R_1	h of R_2	h of R_3	h of R_4	h of R_5	h of R_6	h of R_7	h of R_8
Length (ft)	0	28	50	62	60	55	51	30	3

$$A \approx R_8 = (10)(28) + (10)(50) + (10)(62) + (10)(60) + (10)(55) + (10)(51) + (10)(30) + (10)(3) = 3390\ ft^2$$

$$A \approx 3390\ ft^2$$

(d) For the trapezoidal sum, we are using 8 trapezoids, each height will be 10 ft.

	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
Length (ft)	0	28	50	62	60	55	51	30	3

$$A \approx T_8 = \frac{(10)(0+28)}{2} + \frac{(10)(28+50)}{2} + \frac{(10)(50+62)}{2} + \frac{(10)(62+60)}{2} + \frac{(10)(60+55)}{2} + \frac{(10)(55+51)}{2} + \frac{(10)(51+30)}{2} + \frac{(10)(30+3)}{2} = 3375\ ft^2$$

$$A \approx 3375\ ft^2$$

Riemann Sums PRACTICE

25. For each problem, approximate the area under the given function using the specified number of rectangles/ trapezoids. You are to do all 4 methods to approximate the areas.

#	Function	Interval	Number	Left Rectangles	Right Rectangles	Midpoint Rectangles	Trapezoids
1	$f(x)=x^2-3x+4$	[1,4]	6				
2	$f(x)=\sqrt{x}$	[2,6]	8				
3	$f(x)=2^x$	[0,1]	5				
4	$f(x)=\sin x$	[0,π]	8				

Answers are below:

#	Left Rectangles	Right Rectangles	Midpoint Rectangles	Trapezoids
1	9.125	12.125	10.438	10.625
2	7.650	8.168	7.914	7.909
3	1.345	1.545	1.442	1.445
4	1.974	1.974	2.013	1.974

26. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below. Assuming that Roger's speed is always decreasing, estimate the distance that Roger ran in a) the first half hour and b) the entire race. (Trapezoids)

Time spent running (min)	0	15	30	45	60	75	90
Speed (mph)	12	11	10	10	8	7	0

27. Coal gas is produced at a gasworks. Pollutants in the air are removed by scrubbers, which become less and less efficient as time goes on. Measurements are made at the start of each month (although some months were neglected) showing the rate at which pollutants in the gas are as follows. Use trapezoids to estimate the total number of tons of coal removed over 9 months.

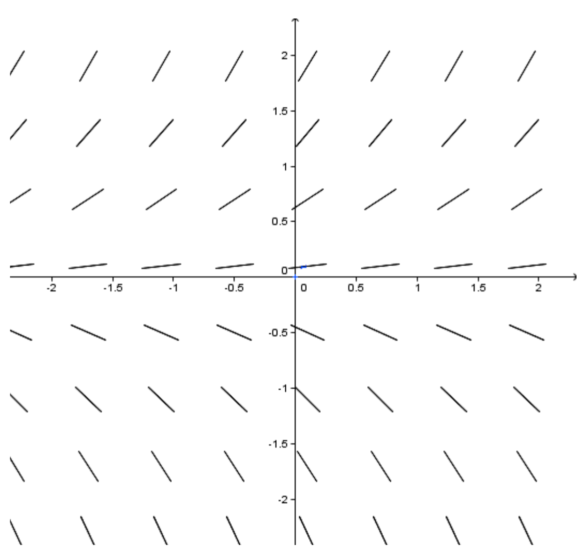
Time (months)	0	1	3	4	6	7	9
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

Another way to get information about our original function, especially if we cannot integrate to get the actual function, is to use the derivative to draw the slopes at various points on the graph and create what is known as a **<u>slope field</u>** (or direction field). We can then start at an initial condition and follow the slope pieces to approximate the solution curve. For now, we'll just focus on creating the slope field. To create a slope field, you use the differential equation given (derivative of the function) and calculate the slope at a number of (*x*, *y*) values. At each of those points on the graph you will draw a small segment with that slope value. So, for instance, if at (1, 1) you calculate that your slope is $-\frac{1}{2}$, at the point (1, 1) on the slope field graph you would draw a small segment with a slope of $-\frac{1}{2}$, and so on until you fill the slope field. This can be a time consuming process, and usually you are not asked to complete this process for more than 10 points on the exam.

<u>Ex. 6 – Making a slope field.</u>

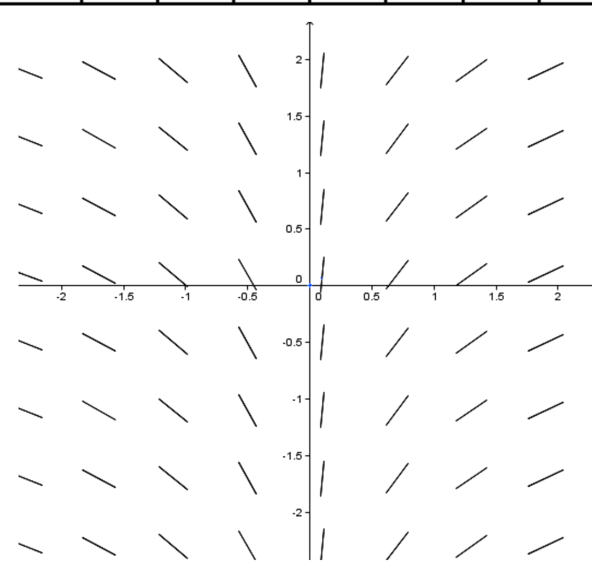
(a) $\frac{dy}{dx} = y$. Fill in the chart for $\frac{dy}{dx}$

(x, y)	-3	-2	-1	0	1	2	3
3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
-1	-1	-1	-1	-1	-1	-1	-1
-2	-2	-2	-2	-2	-2	-2	-2
-3	-3	-3	-3	-3	-3	-3	-3



(b) $\frac{dy}{dx} = \frac{1}{x}$ Fill in the chart for $\frac{dy}{dx}$

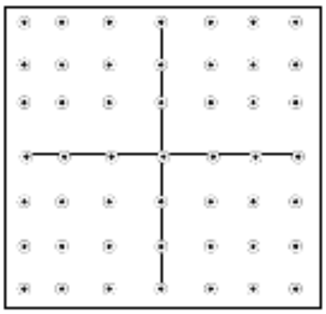
(x, y)	-3	-2	-1	0	1	2	3
3	.-33	-.5	-1	∞	1	.5	.333
2	.-33	-.5	-1	∞	1	.5	333
1	.-33	-.5	-1	∞	1	.5	333
0	.-33	-.5	-1	∞	1	.5	333
-1	.-33	-.5	-1	∞	1	.5	333
-2	.-33	-.5	-1	∞	1	.5	333
-3	.-33	-.5	-1	∞	1	.5	333



Slope Fields PRACTICE

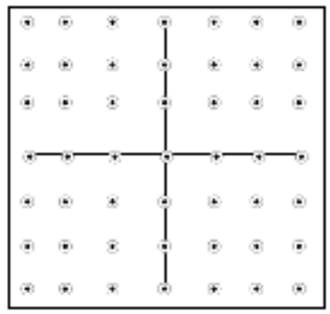
28. $\frac{dy}{dx} = \frac{x}{y}$ Fill in the chart for $\frac{dy}{dx}$.

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							



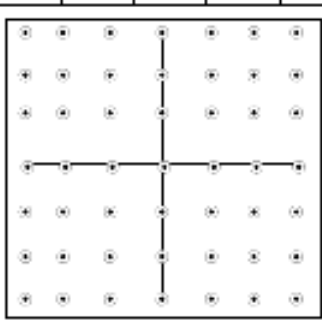
29. $\frac{dy}{dx} = x - y$ Fill in the chart for $\frac{dy}{dx}$

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							



30. $\frac{dy}{dx} = \sin x$ Fill in the chart for $\frac{dy}{dx}$.

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3							



31. $\frac{dy}{dx} = x(1+y)(2-y)$ Fill in the chart for $\frac{dy}{dx}$.

(x, y)	-3	-2	-1	0	1	2	3
3							
2							
1							
0							
-1							
-2							
-3